Teacher Note

Representing Multiplication with Arrays

Representing mathematical relationships is a key element of developing mathematical understanding. In Grade 3, students first encounter multiplication as "groups of" as they brainstorm lists of things that come in equal groups of various sizes and create multiplication situations from those lists. These contexts help them develop visual representations give meaning to multiplication expressions. For example, 5×4 can be visualized as 5 dogs with 4 legs each or 5 rectangles with 4 sides on each rectangle.

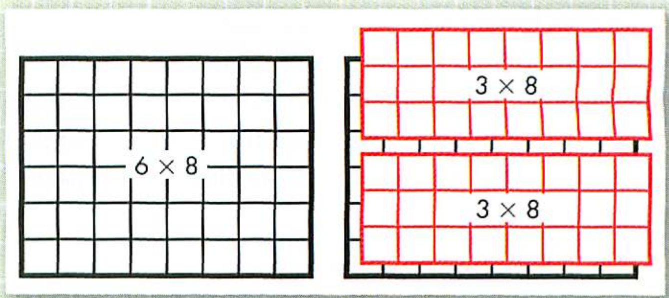
Students extend their understanding of multiplication through their work with arrays. For multiplication, the rectangular array is an important tool. It meets all the criteria for a powerful mathematical representation: it highlights important relationships, provides a tool for solving problems, and can be extended as students apply ideas about multiplication in new areas.

Why Arrays for Multiplication?

Students use rectangular arrays to represent the relationship between a number and its factors: the area of the array is the number, and the length and width of the rectangle are one pair of factors of that number (e.g., a 3×4 rectangle with an area of 12).

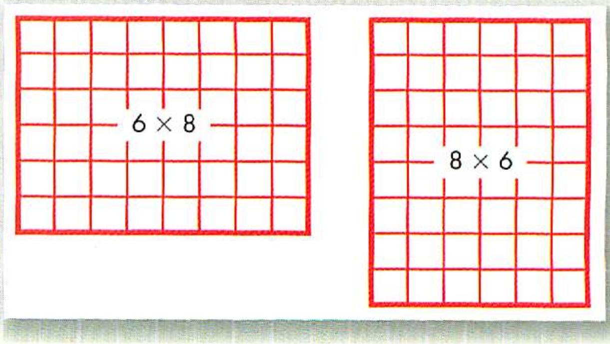
As students come to understand the operation of multiplication in Grades 3 and 4, they gradually move away from thinking of multiplication only as repeated addition. They learn that multiplication has particular properties that distinguish it from addition. Although a number line or 100 chart can show how multiplication can be viewed as adding equal groups, neither of these tools provides easy access to other important properties of multiplication. The rectangular array provides a window into properties that are central to students' work in learning the multiplication combinations and in solving multidigit multiplication and division problems.

For example, suppose that students are working on one of the more difficult multiplication combinations, 6×8. They might think of splitting this multiplication in this way: $6 \times 8 = (3 \times 8) + (3 \times 8)$. In doing so, they are using the distributive property of multiplication, which can be represented by using an array.



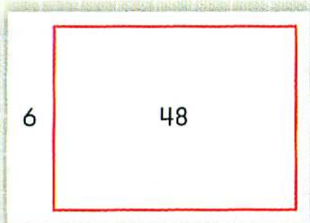
The number that you break up is distributed into parts that must all be multiplied by the other number. The array shows how one dimension of the rectangle (6) is split into parts $(3 + 3)$ to form two new rectangles, each with the dimensions 3×8. This property of multiplication is at the core of almost all common strategies used to solve multiplication problems.

The rectangular array also makes it clearer why the product of 6×8 is the same as the product of 8×6. The array can be rotated to show that six rows with eight in each row has the same number of squares as eight rows with six in each row. The column on one becomes the row on the other, illustrating the commutative property—the fact that you can change the order of two factors in a multiplication expression without changing the product.

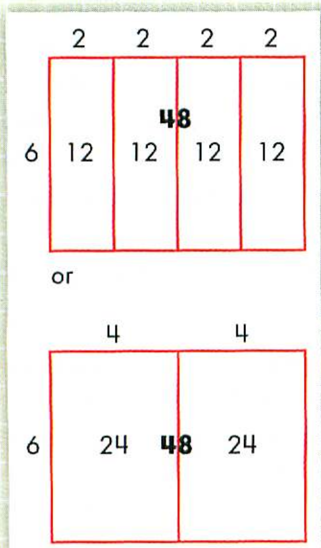


Arrays in this unit serve as a powerful tool for helping students learn multiplication combinations. At first, students often count by the number in a row or column of an array (for example, counting by 8 six times or 6 eight times for the array pictured above). Gradually, they learn to use arrays to visualize how to use known combinations to solve those they are working on ($3 \times 8 = 24$, therefore 6×8 equals twice that amount or 48).

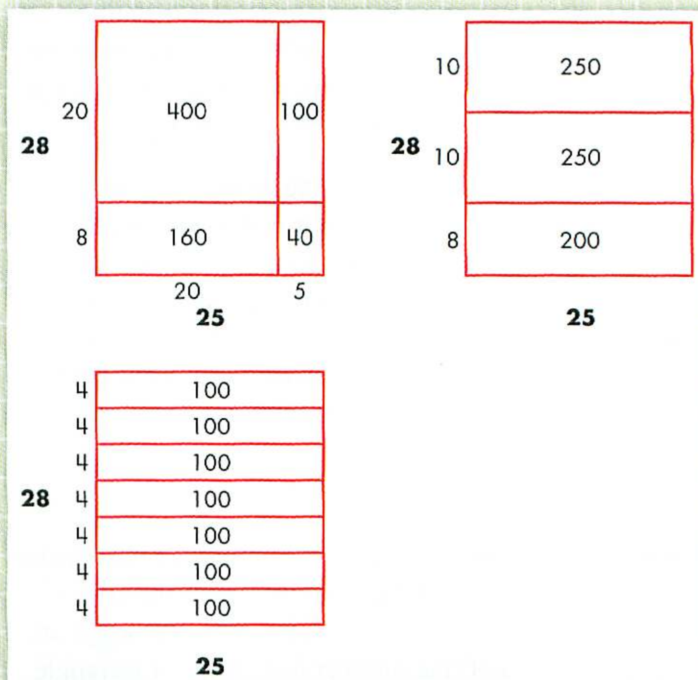
Arrays also support students' learning about the *relationship* between multiplication and division. In a division problem such as $48 \div 6$, the dividend (48) is represented by the number of squares in the array, and the divisor (6) is one dimension of the array. Students develop strategies for division as they find the missing dimension of arrays like the one below.



Students can think of "slicing off" pieces of the rectangle as they gradually figure out the other factor.

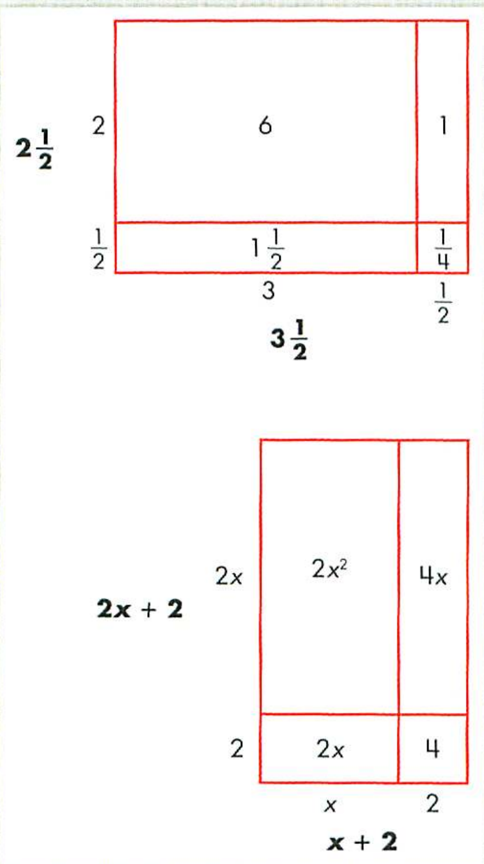


In later grades, arrays are particularly useful for solving or visualizing how to solve multidigit multiplication problems. After students have worked with rectangular arrays for single-digit multiplication combinations and thoroughly understand how an array represents the factors and product, they can use arrays in work later in Grade 4 to solve harder problems. The array for 28×25 can be broken up in many ways.



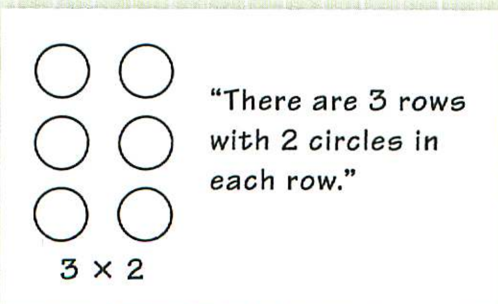
Each of these ways suggests an approach to solving this problem.

Finally, the use of the rectangular array can be extended in later grades as students work with multiplication of fractions and, later, of algebraic expressions.

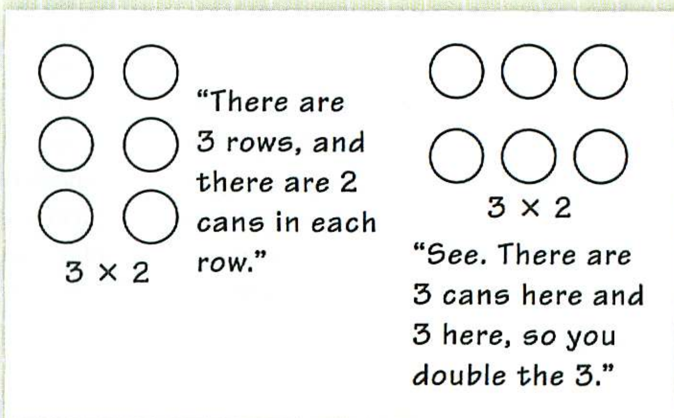


Labeling Arrays

For multiplication notation to describe arrays, the *Investigations* curriculum uses the convention of designating the number of rows first and the number in each row second; for example, 3 × 2 indicates three rows with two in each row.



This convention is consistent with using 3 × 2 to indicate 3 groups of 2 in other multiplication situations (e.g., 3 pots with 2 flowers in each pot). However, it is not necessary or useful to spend time getting students to follow this system rigidly; trying to remember which number stands for rows and which for the number in a row can be unnecessarily distracting for students. When students suggest a multiplication expression for an array, what is important is that they understand what the numbers mean; for example, a student might show how 3 × 2 represents 3 rows of cans with 2 in each row or 3 cans in each of 2 rows.



Note that in other cultures, conventions about interpreting multiplication expressions differ. In some countries, the convention for interpreting 3 × 2 is not "3 groups of 2" but "3 taken 2 times."